A Simulation Study of Drying Kinetics for a Natural Rubber Sheet by Convective Drying

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Abstract
In this work, the drying equipments used convective flow, and the heat source is liquid petroleum gas (LPG). Inside the drying chamber, five samples of natural rubber sheets were used. The weight and temperature were controlled by two spring balances and thermocouples, respectively. The controlled temperatures inside the drying chamber for this study are 40, 50, and 60°C. The experimental data were fitted by using different empirical mathematical models for finding an important parameter named the effective diffusion coefficients at each control temperature. After that a 2D finite elements method (FEM) simulation of drying kinetics with convective drying for a natural rubber sheet was performed. The simulation results fit and predict experimental results well, for moisture ratio of dried natural rubber sheets. However, in the case of temperature, the simulation results have instability in the first period of drying times 0-10 hr, due to the rapidly decreasing moisture ratio in this period. Then, the simulation results go to a steady state. They fit the experimental results well.

Keywords: Natural rubber sheet drying; Convective drying; and Drying kinetics.

1. Introduction

Drying is the most important process in many manufacturing processes. The main aim of this drying is to reduce the moisture inside materials for keeping good quality and long life time of products. Natural rubber sheet manufacturing at the southern of Thailand produced natural rubber of about 3.7 million tons in year 2012 [1]. They use a drying process with hot-air convection to reduce the initial moisture inside a natural rubber sheet. Different heat sources are selected, which depends on area, ease of finding, the cost of each source, and technique. For examples, W. Suchonpanit, et al. [2] studied the drying experiment of rubber sheet by using infrared radiation, hot air, green house, natural convention, open sun, and natural convention drying.

Nowaday, computer simulation play an important role to couple theoretical and experimental science, and engineering. It makes scientists and engineers understand many sophistical phenomena, but some important parameters for use as initial or boundary conditions are very difficult to define. The best method to define these parameters uses empirical mathematical models to fit the experimental results. Hence, the objectives of this study are to quantify experimental equilibrium moisture content of a natural rubber sheet, and choose suitable empirical mathematical models for fitting.
equilibrium moisture contents in terms of the moisture diffusion coefficients. Finally, they are used as initial parameters for simulation software to predict experimental results.

2. Materials and methods

2.1 Drying equipment

Drying equipment for studying the drying kinetics of a rubber sheet is shown in Fig. 1. For investigation the initial moisture content of rubber sheets, small pieces of a square rubber sheet, sized 1x1cm were cut and dried in a hot air oven at 130 °C. Next, five rubber sheets were hung in the drying chamber as shown in Fig.1. The weight of rubber sheets were recorded by using two hook balances. A heat source from LPG was supplied to the drying chamber by using hot air convective flow along an aluminum duct. Hot air was circulated in the system by a circulating fan. The control temperatures in the drying chamber were 40, 50, and 60 °C.

2.2 Drying kinetic and mathematical models

A comparison of moisture ratio between experimental and predicted results at drying temperatures of 40, 50, and 60 °C was performed by the best empirical mathematical models [3], namely Weibull, and Modified Henderson and Pabis:

\[ MR = a - b \exp\left(-kt^*\right) \]  
(1)

\[ MR = a \exp(-kt) + b \exp(-gt) + c \exp(-ht) \]  
(2)

![Fig. 1. Schematic diagram of the convective hot air drying equipment for rubber sheet.](image)
For analysis, the mathematical modeling of thin layer drying equations of rubber sheet predicted the evolution of moisture ratio versus drying time. These empirical models were developed from experimental results using the statistical nonlinear regression and the best fitting; therefore, the goodness of fit was evaluated using the determination coefficient ($R^2$). So, the experimental and empirical mathematical fitting of the moisture ratio ($MR$) in drying rubber sheet of temperature 40, 50, and 60 °C are presented in Fig. 2 and the results of statistical analysis are shown in Table 1.

**Fig. 2.** The drying kinetic of moisture ration ($MR$) as a function of the drying time. The simulation results are carried out by Weibull and Modified Handerson Pabis models, compared to the experimental results at the control temperatures (a) 40°C, (b) 50°C, and (c) 60°C, respectively.
Table 1. Results of statistical analysis on the modeling of rubber sheet with temperatures 40, 50 and 60 °C.

| Temperature | Weibull | Modified Henderson and Pabis |  
|-------------|---------|-------------------------------|---|
|             | $a=0.2806$ | $a=0.2730$ |  
|             | $b=-0.7283$ | $b=0.2476$ |  
|             | $k=0.0056$ | $c=0.4766$ |  
|             | $n=0.8198$ | $g=0.0002$ |  
| 40 °C       | $0.9847$  | $h=0.0032$ |  
|             |          | $k=0.0002$ |  
|             | $a=0.2121$ | $a=0.1427$ |  
|             | $b=-0.7808$ | $b=0.1122$ |  
|             | $k=0.0011$ | $c=0.7909$ |  
|             | $n=1.204$  | $g=0.0001$ |  
| 50 °C       | $0.9849$  | $h=0.0038$ |  
|             |          | $k=0.0013$ |  
|             | $a=0.1902$ | $a=0.1514$ |  
|             | $b=-0.8556$ | $b=0.1189$ |  
|             | $k=0.0017$ | $c=0.8348$ |  
|             | $n=1.1702$  | $g=0.0002$ |  
| 60 °C       | $0.9815$  | $h=0.0053$ |  
|             |          | $k=0.0002$ |  

2.3 The effective moisture diffusivity of rubber sheet

In this section, the effective moisture diffusivity ($D_{eff}$) is determined from the experimental drying curve at the control temperatures. The governing equation is described by Fick’s law for the moisture concentration in biological materials, which can be expressed as:

$$
\frac{\partial c}{\partial t} = \nabla \left[ -D_{eff} \nabla c \right]
$$

(3)
The assumptions for this study are based on uniform initial temperature gradients, negligible external resistance, temperature gradients, shrinkage during drying, and constant diffusion coefficient. By applying the thin layer drying, the infinite slab sheet model is used, in which the thickness is smaller than the length of slab sheet and drying time is large. So, the approximate solution [3] of Eq. 4 under these conditions is:

\[
\ln(MR) = \ln\left(\frac{8}{\pi^2}\right) - \frac{\pi^2 D_{eff} t}{L^2}
\]

where, \(t\) is drying time, and \(L\) is thickness of the slab sheet (3mm). Therefore, the effective diffusivity can be calculated from the slope of the plot \(\ln(MR)\) vs. time. From Table. 1, the best fitting comes from Modified Handerson and Pabis, which gives a higher \(R^2\) than the Weibull. Therefore, the results of the effective diffusivity of rubber sheet, which fitted the experimental results at the control temperatures of 40, 50, and 60°C are 2.503x10^{-10}, 2.948x10^{-10}, and 4.513x10^{-10} m^2/hr, respectively.

3. Simulation results and discussions

After the effective diffusivity is obtained, a simulation study of the convection drying of a rubber sheet is carried out by FEM simulation software, COMSOL Multiphysics™. This simulation also models the moisture concentration in the rubber sheet. In this regard, drying yield is a quantity that measures how much moisture, in percent, remains in the rubber sheet after a drying process. Moreover, the moisture concentration also influences heat loss due to vaporization and also by changing the rubber sheet’s thermal conductivity. This model couples time-dependent interfaces, describing the temperature and moisture concentration. The simulation does not model the convective velocity field outside the rubber sheet because the coefficients for convective heat and moisture transfer to the surrounding air are given. The model defines the geometry of the rubber sheet in 2-dimensions. The width is 3.0 mm and the length is 40.0 cm. The physics of this model is composed of the General Heat Transfer (GHT) module and Diffusion module. For the GHT module, the governing equation is:

\[
\rho c_p \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = 0
\]

where, \(\rho\) is the density (917kg/m^3) [4], \(c_p\) is specific heat capacity (J/kg.K) [4], and \(k\) is thermal conductivity (W/m.K). Assuming symmetry for the temperature and air convection adds heat on all boundaries, the initial temperature of rubber sheet is 27°C. Also, the boundary conditions for the heat transfer interface is shown as:

\[
\hat{n}_i (k \nabla T) = h_f (T_{inf} - T) + \hat{n}_i (D_m \lambda \nabla c)
\]
where, $h_s$ is the heat transfer coefficient (10 W/m$^2$.K), $T_{in}$ is the drying chamber air temperature (50°C), $D_m$ is the surface moisture diffusion coefficient (5x10^{-10} m$^2$/s), and $\lambda$ is the molar latent heat of vaporization (2.3x10^{4} J/mol [5]). The vaporization of water at the rubber sheet’s outer boundaries generates a heat flux out of the sheet. This heat flux is represented by the second term of Eq. (6).

For the diffusion module, the governing equation is shown in Eq. (7), by assuming the initial moisture concentration is 78% of rubber sheet density and $D_{eff}$ at the control temperatures of 40, 50, and 60°C are 2.503x10^{-10}, 2.948x10^{-10}, and 4.5126x10^{-10} m$^2$/hr, respectively. The boundary conditions for the diffusion are:

$$\vec{n} \cdot (D_{eff} \nabla c) = k_s(c_b - c) \quad (7)$$

Where, $k_s$ is mass transfer coefficient ($k_s = h_s / \rho \varepsilon_s$, m/s), $h_m$ is the mass transfer coefficient in mass unit (1.67x10^{-7} kg/m$^2$.s), $\varepsilon_s$ is the specific moisture capacity (6x10^{-3} kg moisture/kg rubber sheet), $c_b$ is air (bulk) moisture concentration (18.34 kg/m$^3$), and $c$ is the moisture concentration in the rubber sheet.

The simulation results of drying kinetics are compared to the experimental results of rubber sheet at the control temperatures 40, 50, and 60°C, as shown in Fig. 3 (a), (b), and (c), respectively. In these figures, the simulation can picks up trends of the experimental normalized moisture ratio. However, the simulation results of temperature yield over-predict experiment in the first period or transient state around 0-10 hr, after that, the simulation results go to a steady state. In this period, the simulation results match the experimental results very well, because the lines of simulation pass through the error bar of each experiment data points except the case of control temperature at 40°C. It cannot predict this temperature. For this case, the average of the control temperature shifts up to 43°C. The transient phenomenon comes from the characteristics of drying kinetics of moisture ratio in the first period, because a natural rubber sheet is a hygroscopic material. In the first drying period of around 0-10hr, the normalized moisture ratio in the rubber sheet rapidly decreases. This evidence is shown in Fig. 3 for every control temperature. It causes a predict temperature over-shoot in this period, when the normalized moisture ratio slowly reduces in the steady state period. The simulation results match experimental results in this period. It should be noted that the drying time for a control temperature of 60°C is shorter than 40 and 50°C because it has more energy to force moisture inside rubber sheets to move out. Then, the normalized moisture ratio reaches target quickly, so the drying time is short.

4. Conclusions

The best empirical mathematical model, Modified Handerson and Pabis, was used to fit the experimental data of a drying natural rubber sheet. The effective moisture diffusivities were calculated at the control temperatures of 40, 50, and 60°C which are 2.503x10^{-10}, 2.948x10^{-10}, and 4.514x10^{-10} m$^2$/hr, respectively. These important parameters were used as an initial condition to study the drying kinetics of a drying natural rubber sheet. The simulation results were carried out by simulation software which is based on the 2-dimensional finite elements method. For the case of moisture ratio, the simulation results can predict the trend of experimental results very well. However, in the case of temperature, the simulation results show the instability in the first drying period, due to the rapidly decreasing normalized moisture ratio.
Fig. 3. Normalized moisture ratio (top panel) and temperature (bottom panel) of dried rubber sheet as a function of drying time. Experimental results are compared to simulation results, which are carried out by COMSOL Multiphysics™ for the control temperatures (a) 40°C, (b) 50°C, and (c) 60°C.

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References


